PENGOLAHAN SINYAL DIGITAL (PSD)

- Transformasi-Z,
- Invers Trans-Z,
- Representasi Sistem (Fungsi Transfer, Respon Impuls, Persamaan selisih, Struktur Realisasi)
- Komponen dan Sifat Sistem,
- Analisis Kestabilan Sistem

Pengampu Matakuliah

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Prodi S1-Sistem Komputer

Aturan Perkuliahan

- Keterlambatan kehadiran maksimal 15 menit setelah kehadiran dosen.
- Keterlambatan pengumpulan tugas akan mengurangi nilai tugas.
- Bobot penilaian:
  - Tugas 20%, Kuis 15%
  - UTS 30%, UAS 35%

Preview

- Apa itu Transformasi-Z ?
- Kenapa/Kapan kita menggunakan transformasi-Z
- Apa itu Invers Transformasi-Z ? Kapan digunakan?
- Apa yang anda ketahui tentang sistem ?

TRANSFORMASI-Z

Latar Belakang

"Domains of representation "

Domain-n (discrete time) :
Sequence, impulse response, persamaan beda

Domain- ω :
Freq. response, spectral representation

Domain-z :
Operator, dan pole-zero

Apabila suatu kasus sulit dipecahkan pada suatu domain tertentu, maka transformasi ke domain yang lain akan mudah menyelesaikannya.

Transformasi-Z Langsung

- Transformasi-Z sinyal waktu diskrit $x(n)$ didefinisikan sebagai deret pangkat:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

- Dimana z adalah variable kompleks.
- Karena transformasi-z adalah deret pangkat tak berhingga, transformasi hanya berlaku untuk nilai-nilai z yang deretnya konvergen (jumlah dari deret berhingga).
- Daerah konvergensi (ROC) adalah himpunan nilai z agar X(Z) nilainya berhingga.

Contoh 1

Tentukan transformasi-Z dari beberapa sinyal di bawah ini:

a. $x_1(n) = \delta(n)$

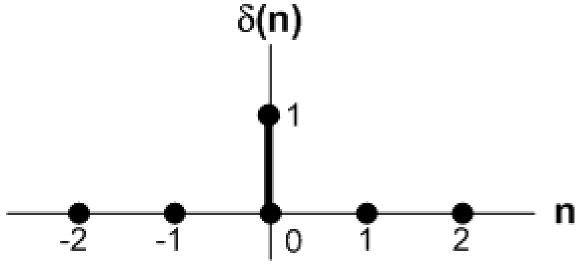
b. $x_2(n) = \delta(n-k),\ k > 0$

c. $x_3(n) = \delta(n+k),\ k > 0$

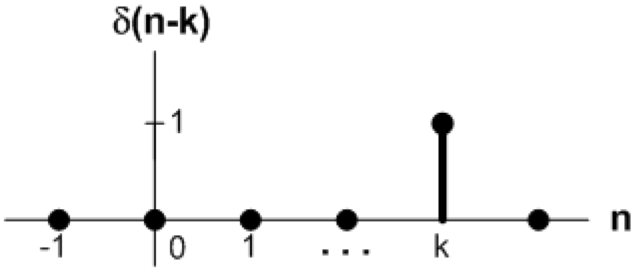
Solusi Contoh 1

Jawab:

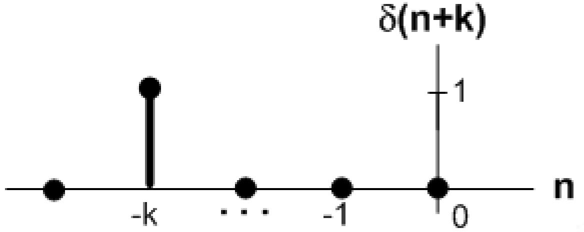
a. $X_1(z) = \sum_{n=-\infty}^{\infty} \delta(n) z^{-n} = 1 \, . \, z^0 = 1$



b. $X_2(z) = \sum_{n=-\infty}^{\infty} \delta(n-k) z^{-n} = z^{-k}$



c. $X_3(z) = \sum_{n=-\infty}^{\infty} \delta(n+k) z^{-n} = z^{k}$



Contoh 2

- $x_1(n) = \{\underset{\uparrow}{1}, 2, 5, 7, 0, 1\}$

$$X_1(z) = 1 + 2z^{-1} + 5z^{-2} + 7z^{-3} + z^{-5}$$

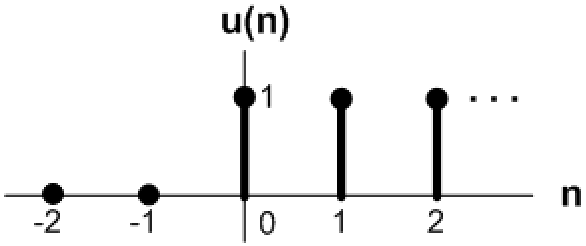
- $x_2(n) = \{1, 2, \underset{\uparrow}{5}, 7, 0, 1\}$

$$X_1(z) = 1z^{2} + 2z^{1} + 5 + 7z^{-1} + z^{-3}$$

Contoh 3

- Tentukan transformasi-Z dari sinyal $x(n) = u(n)$

Jawab:



$$X(z) = \sum_{n=0}^{\infty} u(n)\, z^{-n} = 1 + z^{-1} + z^{-2} + \ldots$$

$$= \frac{1}{1 - z^{-1}} \text{ , dimana } \left| z^{-1} \right| < 1 \rightarrow ROC: |z| > 1$$

$$\therefore \quad x(n) = u(n) \quad \rightarrow \quad X(z) = \frac{1}{1 - z^{-1}} \quad , ROC: |z| > 1$$

Masih ingat penulisan deret geometri?

$$a_0 + a_0.r + a_0.r^2 + a_0.r^3 + \ldots = \frac{a_0}{1-r};\ |r|<1$$

Suku pertama

Suku kedua

Rasio = suku kedua **dibagi** suku pertama

Rumus deret geometri

ROC = Region of convergence

Apa bedanya dengan deret aritmatika?

Apa ciri-ciri deret geometri?

Contoh 4

- Tentukan transformasi-Z dari sinyal $x(n) = \alpha^n u(n)$

Jawab:

$$X(z) = \sum_{n=0}^{\infty} \alpha^n u(n) z^{-n} = \sum_{n=0}^{\infty} \left(\alpha z^{-1}\right)^n$$

$$= \sum_{n=0}^{\infty} (A)^n = 1 + A + A^2 + A^3 + \ldots = \frac{1}{1-A}$$

$$= \frac{1}{1-\alpha z^{-1}} \text{, dimana } \left|\alpha z^{-1}\right| < 1 \rightarrow ROC: |z| > |\alpha|$$

$$\therefore \quad x(n) = \alpha^n u(n) \quad \rightarrow \quad X(z) = \frac{1}{1-\alpha z^{-1}} \quad , ROC: |z| > |\alpha|$$

Tabel Fungsi Dasar TZ

Sequence	z-Transform	Region of Convergence
$\delta(n)$	1	all z
$\alpha^n u(n)$	$\dfrac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert > \lvert\alpha\rvert$
$-\alpha^n u(-n-1)$	$\dfrac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert < \lvert\alpha\rvert$
$n\alpha^n u(n)$	$\dfrac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
$-n\alpha^n u(-n-1)$	$\dfrac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert < \lvert\alpha\rvert$
$\cos(n\omega_0)u(n)$	$\dfrac{1-(\cos\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$\sin(n\omega_0)u(n)$	$\dfrac{(\sin\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$

Sifat-Sifat TZ

Property	Sequence	z-Transform	Region of Convergence
Linearity	$ax(n) + by(n)$	$aX(z) + bY(z)$	Contains $R_x \cap R_y$
Shift	$x(n - n_0)$	$z^{-n_0}X(z)$	R_x
Time reversal	$x(-n)$	$X(z^{-1})$	$1/R_x$
Exponentiation	$\alpha^n x(n)$	$X(\alpha^{-1}z)$	$\|\alpha\|R_x$
Convolution	$x(n) * y(n)$	$X(z)Y(z)$	Contains $R_x \cap R_y$
Conjugation	$x^*(n)$	$X^*(z^*)$	R_x
Derivative	$nx(n)$	$-z\dfrac{dX(z)}{dz}$	R_x

SIFAT-SIFAT TRANSFORMASI-Z

- **Linieritas**

$$x(n) = a\,x_1(n) + b\,x_2(n) \quad \rightarrow \quad X(z) = a\,X_1(z) + b\,X_2(z)$$

- **Contoh 5:**

Tentukan transformasi Z dari sinyal $x(n) = \left[3(2)^n - 4(3)^n\right]u(n)$

Jawab:

$$x_1(n) = (2)^n u(n) \rightarrow X_1(z) = \frac{1}{1-2z^{-1}} \quad , ROC: |z| > 2$$

$$x_2(n) = (3)^n u(n) \rightarrow X_2(z) = \frac{1}{1-3z^{-1}} \quad , ROC: |z| > 3$$

$$x(n) = \left[3(2)^n - 4(3)^n\right]u(n) \rightarrow X(Z) = \frac{3}{1-2z^{-1}} - \frac{4}{1-3z^{-1}} = \frac{-1-z^{-1}}{1-5z^{-1}+6z^{-2}}$$

$$ROC: |z| > 2 \cap |z| > 3 \rightarrow ROC: |z| > 3$$

SIFAT-SIFAT TRANSFORMASI-Z

- **Pergeseran**

$$x(n-n_0) \quad \rightarrow \quad z^{-n_0} X(Z)$$

- **Contoh 5:**

Tentukan transformasi Z dari sinyal $x(n) = u(n-3)$

Jawab:

$$x_1(n) = u(n) \rightarrow X_1(Z) = \frac{1}{1-z^{-1}}, ROC : R_x = |z| > 1$$

$$\therefore \; x(n) = u(n-3) \rightarrow X(Z) = z^{-3} X_1(Z) = \frac{z^{-3}}{1-z^{-1}}, \; ROC : R_x = |z| > 1$$

SIFAT-SIFAT TRANSFORMASI-Z

- **Time Reversal**

$$x(-n) \quad \rightarrow \quad X(z^{-1})$$

- **Contoh 6:**

Tentukan transformasi Z dari sinyal $x(n) = u(-n)$

Jawab:

$$x_1(n) = u(n) \rightarrow X_1(Z) = \frac{1}{1 - z^{-1}}, ROC: R_x = |z| > 1$$

$$\therefore \; x(n) = u(-n) \rightarrow X(z) = \frac{1}{1 - \left(z^{-1}\right)^{-1}} = \frac{1}{1 - z}, \; ROC: \; 1/R_x = |z| < 1$$

SIFAT-SIFAT TRANSFORMASI-Z

- **Diferensiasi dalam domain z**

$$nx(n) \rightarrow -z\frac{dX(z)}{dz}$$

$$\frac{u}{v}\ turunannya = \frac{u'.v - u.v'}{v^2}$$

- **Contoh 7:**

Tentukan transformasi Z dari sinyal $x(n) = n\,a^n u(n)$

Jawab:

$$x_1(n) = a^n u(n) \rightarrow X_1(z) = \frac{1}{1-az^{-1}}, ROC : R_x = |z| > a$$

$$\therefore x(n) = n\,a^n u(n) \rightarrow X(z) = -z\frac{dX_1(z)}{dz} = -z\frac{d}{dz}\left(\frac{1}{1-az^{-1}}\right)$$

$$= (-z)\frac{-az^{-2}}{\left(1-az^{-1}\right)^2} = \frac{az^{-1}}{\left(1-az^{-1}\right)^2}$$

$$\therefore n\,a^n u(n) \rightarrow \frac{az^{-1}}{\left(1-az^{-1}\right)^2}$$

SIFAT-SIFAT TRANSFORMASI-Z

- **Konvolusi antara dua sinyal**

$$x(n) = x_1(n) * x_2(n) \quad \rightarrow \quad X(z) = X_1(z) X_2(z)$$

- **Contoh 8:**

Tentukan konvolusi antara $x_1(n)$ dan $x_2(n)$ dengan :

$$x_1(n) = \{1, -2, 1\} \qquad x_2(n) = \begin{cases} 1, & 0 \le n \le 5 \\ 0, & lainnya \end{cases}$$

Jawab:

$$X_1(z) = 1 - 2z^{-1} + z^{-2} \qquad X_2(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5}$$

$$X(z) = X_1(z) X_2(z) = (1 - 2z^{-1} + z^{-2})(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5})$$

$$X(z) = X_1(z) X_2(z) = 1 - z^{-1} - z^{-6} + z^{-7}$$

$$\therefore\ x(n) = x_1(n) * x_2(n) = \{1, -1, 0, 0, 0, 0, -1, 1\}$$

Latihan Transformasi-Z

- Ubahlah persamaan di bawah ini menjadi domain Z!
- $3^{n}.U(n+3)$
- $n^{2}.U(n)$
- $2^{-n}.U(n-1)$
- $10^{-n}.(U(n)-U(n-2))$

INVERS TRANSFORMASI-Z

INVERS TRANSFORMASI-Z

- **Definisi invers transformasi-Z**

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad \longrightarrow \quad x(n) = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz$$

Teorema residu Cauchy :

$$\frac{1}{2\pi j} \oint_C \frac{f(z)}{(z-z_o)^k} dz = \begin{cases} \dfrac{1}{(k-1)!} \dfrac{d^{k-1} f(z)}{dz^{k-1}} \bigg|_{z=z_o} & , bila\ z_o\ di\ dalam\ C \\ 0, & bila\ z_o\ di\ luar\ C \end{cases}$$

- **Ekspansi deret dalam z dan z^{-1}**

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

- **Contoh 11:**

Tentukan invers transformasi-z dari $X(z) = \dfrac{1}{1 - \frac{3}{2} z^{-1} + \frac{1}{2} z^{-2}}$

Jawab:

$$X(z) = 1 + \frac{3}{2} z^{-1} + \frac{7}{4} z^{-2} + \frac{15}{8} z^{-3} + \frac{31}{16} z^{-4} \cdots$$

$$\therefore \; x(n) = \left\{ \underset{\uparrow}{1}, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \cdots \right\}$$

- **Ekspansi fraksi-parsial dan tabel transformasi-z**

$$X(z) = \alpha_1 X_1(z) + \alpha_2 X_2(z) + \cdots + \alpha_K X_K(z)$$

$$x(n) = \alpha_1 x_1(n) + \alpha_2 x_2(n) + \cdots + \alpha_K x_K(n)$$

- **Contoh 12:**

Tentukan invers transformasi-z dari $X(z) = \dfrac{1}{1 - 1{,}5z^{-1} + 0{,}5z^{-2}}$

Jawab:

$$X(z) = \frac{z^2}{z^2 - 1{,}5z + 0{,}5} = \frac{z^2}{(z-1)(z-0{,}5)}$$

$$\frac{X(z)}{z} = \frac{z}{z^2 - 1{,}5z + 0{,}5} = \frac{A_1}{(z-1)} + \frac{A_2}{(z-0{,}5)}$$

$$\frac{X(z)}{z}=\frac{z}{z^2-1,5z+0,5}=\frac{A_1}{(z-1)}+\frac{A_2}{(z-0,5)}$$

$$=\frac{A_1(z-0,5)+A_2(z-1)}{(z-1)(z-0,5)}=\frac{(A_1+A_2)z-(0,5A_1+A_2)}{z^2-1,5z+0,5}$$

$$\frac{X(z)}{z}=\frac{z}{z^2-1,5z+0,5}=\frac{(A_1+A_2)z-(0,5A_1+A_2)}{z^2-1,5z+0,5}$$

$$A_1+A_2=1 \quad 0,5A_1+A_2=0 \quad \rightarrow \quad A_2=-0,5A_1$$

$$A_1-0,5A_1=0,5A_1=1 \quad \rightarrow \quad A_1=2 \quad \rightarrow \quad A_2=-1$$

$$X(z)=\frac{2}{(1-z^{-1})}-\frac{1}{(1-0,5z^{-1})} \quad \Longrightarrow \quad \therefore \; x(n)=[2-(0,5)^n]u(n)$$

- **Pole-pole berbeda semua**

$$\frac{X(z)}{z} = \frac{A_1}{z - p_1} + \cdots + \frac{A_k}{z - p_k} + \cdots + \frac{A_N}{z - p_N}$$

$$\frac{(z - p_k)X(z)}{z} = \frac{(z - p_k)A_1}{z - p_1} + \cdots + A_k + \cdots + \frac{(z - p_k)A_N}{z - p_N}$$

$$\left.\frac{(z - p_k)X(z)}{z}\right|_{z = p_k} = A_k$$

Contoh Soal 8.17

Tentukan zero-state response dari suatu sistem LTI yang mendapat input x(n) = u(n) dan dinyatakan oleh persamaan beda :

$$y(n) + 6y(n-1) + 8y(n-2) = 5x(n) - 28x(n-1) + 8x(n-2)$$

Jawab:

$$Y(z) + 6z^{-1}Y(z) + 8z^{-2}Y(z) = 5X(z) - 28z^{-1}X(z) + 8z^{-2}X(z)$$

$$X(z) = \frac{1}{1-z^{-1}} \longrightarrow Y(z) = \frac{(5 - 28z^{-1} + 8z^{-2})}{1 + 6z^{-1} + 8z^{-2}} \frac{1}{1-z^{-1}}$$

$$\frac{Y(z)}{z} = \frac{(5z^2 - 28z + 8)}{(z^2 + 6z + 8)(z-1)} = \frac{A_1}{z+2} + \frac{A_2}{z+4} + \frac{A_3}{z-1}$$

$$\frac{Y(z)}{z} = \frac{(5z^2 - 28z + 8)}{(z+2)(z+4)(z-1)} = \frac{A_1}{z+2} + \frac{A_2}{z+4} + \frac{A_3}{z-1}$$

$$A_1 = \frac{(z+2)Y(z)}{z} = \left.\frac{5z^2 - 28z + 8}{(z+4)(z-1)}\right|_{z=-2} = \frac{20 + 56 + 8}{(2)(-3)} = \frac{84}{-6} = -14$$

$$A_2 = \frac{(z+4)Y(z)}{z} = \left.\frac{5z^2 - 28z + 8}{(z+2)(z-1)}\right|_{z=-4} = \frac{80 + 112 + 8}{(-2)(-5)} = \frac{200}{10} = 20$$

$$A_3 = \frac{(z-1)Y(z)}{z} = \left.\frac{5z^2 - 28z + 8}{(z+2)(z+4)}\right|_{z=1} = \frac{5 - 28 + 8}{(3)(5)} = \frac{-15}{15} = -1$$

$$\frac{Y(z)}{z} = \frac{-14}{z+2} + \frac{20}{z+4} + \frac{-1}{z-1} \qquad Y(z) = \frac{-14}{1 + 2z^{-1}} + \frac{20}{1 + 4z^{-1}} + \frac{-1}{1 - z^{-1}}$$

$$y_{zs}(n) = [-14(-2)^n + 20(-4)^n - 1]u(n)$$

- **Ada dua pole yang semua**

$$\frac{X(z)}{z} = \frac{A_1}{z - p_1} + \cdots + \frac{A_{1k}}{(z - p_k)^2} + \frac{A_{2k}}{z - p_k} + \cdots + \frac{A_N}{z - p_N}$$

$$A_{1k} = \left.\frac{(z - p_k)^2 X(z)}{z}\right|_{z=p_k}$$

$$A_{2k} = \frac{d}{dz}\left[\frac{(z - p_k)^2 X(z)}{z}\right]_{z=p_k}$$

Contoh Soal 8.18

Tentukan transformasi-Z balik dari :

$$X(z) = \frac{1}{(1+z^{-1})(1-z^{-1})^2}$$

Jawab:

$$\frac{X(z)}{z} = \frac{z^2}{(z+1)(z-1)^2} = \frac{A_1}{z+1} + \frac{A_2}{(z-1)^2} + \frac{A_3}{(z-1)}$$

$$A_1 = \frac{(z+1)X(z)}{z} = \left.\frac{z^2}{(z-1)^2}\right|_{z=-1} = \frac{1}{4}$$

$$A_2 = \frac{(z-1)^2 X(z)}{z} = \left.\frac{z^2}{(z+1)}\right|_{z=1} = \frac{1}{2}$$

$$A_3 = \frac{d}{dz}\left[\frac{(z-1)^2 X(z)}{z}\right] = \frac{d}{dz}\left[\frac{z^2}{(z+1)}\right]$$

$$= \frac{(2z)(z+1)-(1)(z^2)}{(z+1)^2} = \left.\frac{z^2+2z}{(z+1)^2}\right|_{z=1} = \frac{3}{4}$$

$$x(n) = [\frac{1}{4}(-1)^n + \frac{1}{2}n + \frac{3}{4}]u(n)$$

- **Pole kompleks**

$$X(z) = \frac{A_1}{1 - p_1 z^{-1}} + \frac{A_2}{1 - p_2 z^{-1}}$$

$$p_1 = p \quad \rightarrow \quad p_2 = p^* \qquad\qquad A_1 = A \quad \rightarrow \quad A_2 = A^*$$

$$\frac{A}{1 - pz^{-1}} + \frac{A^*}{1 - p^* z^{-1}} = \frac{A - Ap^* z^{-1} + A^* - A^* pz^{-1}}{1 - pz^{-1} - p^* z^{-1} + pp^* z^{-2}}$$

$$\frac{(A + A^*) - (Ap^* + A^* p)z^{-1}}{1 - (p + p^*)z^{-1} + pp^* z^{-2}} = \frac{b_o + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$A + A^* = \mathrm{Re}(A) + j\,\mathrm{Im}(A) + \mathrm{Re}(A) - j\,\mathrm{Im}(A) = 2\,\mathrm{Re}(A)$$

$$b_o = A + A^* = 2\,\mathrm{Re}(A)$$

$$p + p^* = \mathrm{Re}(p) + j\,\mathrm{Im}(p) + \mathrm{Re}(p) - j\,\mathrm{Im}(p) = 2\,\mathrm{Re}(p)$$

$$a_1 = (p + p^*) = -2\,\mathrm{Re}(p)$$

$$pp^* = [\mathrm{Re}(p) + j\,\mathrm{Im}(p)][\mathrm{Re}(p) - j\,\mathrm{Im}(p)]$$

$$= \mathrm{Re}^2(p) + \mathrm{Im}^2(p) = |p|^2 \quad \rightarrow \quad a_2 = pp^* = |p|^2$$

$$Ap^* + A^*p = [\mathrm{Re}(A) + j\,\mathrm{Im}(A)][\mathrm{Re}(p) - j\,\mathrm{Im}(p)]$$

$$+ [\mathrm{Re}(A) - j\,\mathrm{Im}(A)][\mathrm{Re}(p) + j\,\mathrm{Im}(p)]$$

$$= 2\,\mathrm{Re}(A)\,\mathrm{Re}(p) + 2\,\mathrm{Im}(A)\,\mathrm{Im}(p)$$

$$Ap^* = [\mathrm{Re}(A) + j\,\mathrm{Im}(A)][\mathrm{Re}(p) - j\,\mathrm{Im}(p)]$$

$$= [\mathrm{Re}(A)\,\mathrm{Re}(p) + \mathrm{Im}(A)\,\mathrm{Im}(p)] + j[\mathrm{Re}(p)\,\mathrm{Im}(A) - \mathrm{Re}(A)\,\mathrm{Im}(p]$$

$$b_1 = -(Ap^* + A^*p) = -2\,\mathrm{Re}(Ap^*)$$

Contoh Soal 8.19

Tentukan invers transformasi-Z dari :

$$X(z) = \frac{1 + z^{-1}}{1 - z^{-1} + 0{,}5z^{-2}}$$

Jawab:

$$X(z) = \frac{1 + z^{-1}}{1 - z^{-1} + 0{,}5z^{-2}} = \frac{b_o + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$b_o = 2\,\mathrm{Re}(A) = 1 \quad \rightarrow \quad \mathrm{Re}(A) = 0{,}5$$

$$a_1 = -2\,\mathrm{Re}(p) = -1 \quad \rightarrow \quad \mathrm{Re}(p) = 0{,}5$$

$$b_1 = 2\,\mathrm{Re}(Ap^*) = 1 \quad \rightarrow \quad \mathrm{Re}(Ap^*) = 0{,}5$$

$$a_2 = |p|^2 = 0{,}5 \quad \rightarrow \quad \mathrm{Re}^2(p) + \mathrm{Im}^2(p) = 0{,}5$$

$$\text{Re}(p) = 0{,}5 \qquad \text{Re}(A) = 0{,}5$$

$$\text{Re}^2(p) + \text{Im}^2(p) = 0{,}25 + \text{Im}^2(p) = 0{,}5$$

$$\text{Im}^2(p) = 0{,}25 \;\rightarrow\; \text{Im}(p) = 0{,}5 \;\rightarrow\; p = 0{,}5 + j\,0{,}5$$

$$Ap^* = [0{,}5 + j\,\text{Im}(A)](0{,}5 - j0{,}5)$$

$$\text{Re}(Ap^*) = 0{,}25 + 0{,}5\,\text{Im}(A) = 0{,}5$$

$$\text{Im}(A) = 0{,}25 \quad\rightarrow\quad A = 0{,}5 + j\,0{,}25$$

$$X(z) = \frac{A}{1 - pz^{-1}} + \frac{A\,*}{1 - p\,*\,z^{-1}}$$

$$= \frac{0{,}5 + j\,0{,}25}{1 - (0{,}5 + j\,0{,}5)z^{-1}} + \frac{0{,}5 - j\,0{,}25}{1 - (0{,}5 - j\,0{,}5)z^{-1}}$$

$$X(z) = \frac{0,5 + j\,0,25}{1 - (0,5 + j\,0,5)z^{-1}} + \frac{0,5 - j\,0,25}{1 - (0,5 - j\,0,5)z^{-1}}$$

$$0,5 + j\,0,5 = 0,707e^{j45} \qquad\qquad 0,5 - j\,0,5 = 0,707e^{-j45}$$

$$x(n) = (0,5 + j\,0,25)(0,707e^{j45})^{n} + (0,5 - j\,0,25)(0,707e^{j45})^{n}$$

$$= (0,5)(0,707)^{n}(\cos 45n + j\sin 45n)$$

$$+ j(0,25)(0,707^{n})(\cos 45n + j\sin 45n)$$

$$+ (0,5)(0,707)^{n}(\cos 45n - j\sin 45n)$$

$$- j(0,25)(0,707^{n})(\cos 45n - j\sin 45n)$$

$$= (0,707)^{n}\cos 45n - 0,5(0,707)^{n}\sin 45n$$

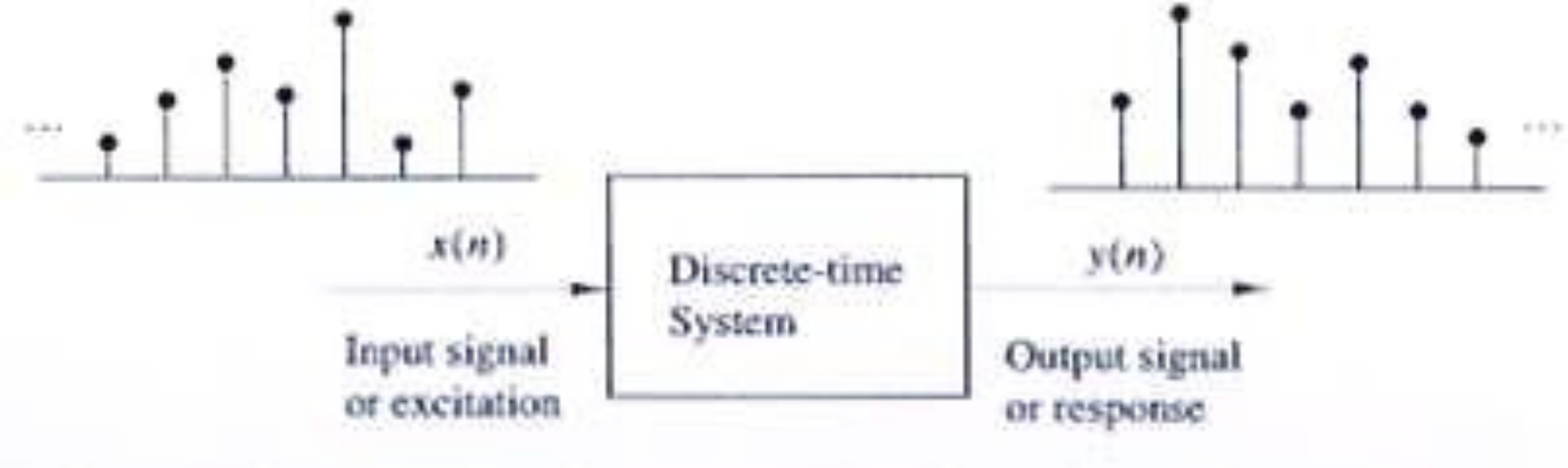
Latihan Invers Transformasi-Z

- Ubahlah persamaan di bawah ini menjadi domain n!
- $\frac{z}{1+z^2}$, dg ekspansi deret
- $\frac{2z}{z^2+4z+3}$, dg ekspansi fraksi-parsial & tabel TZ
- $\frac{2z-2}{Z+1}$, dg ekspansi fraksi-parsial & tabel TZ

SISTEM WAKTU DISKRIT

Deskripsi Input-Output

- Ekspresi matematik :
  - Hubungan antara input dan output
- x(n) = input (masukan, eksitasi)
- y(n) = output (keluaran, respon)
- T = Transformasi (operator)
- Sistem dipandang sebagai black box

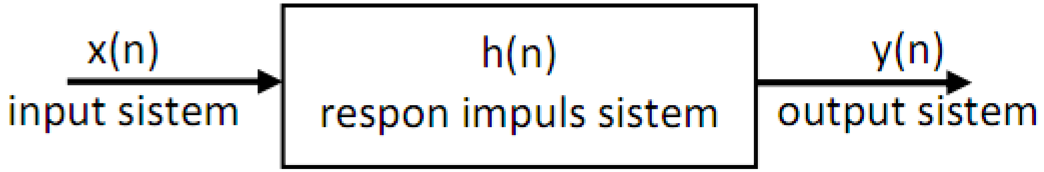


$$y(n) = T[x(n)]$$

$$x(n) \xrightarrow{T} y(n)$$

Representasi Sistem

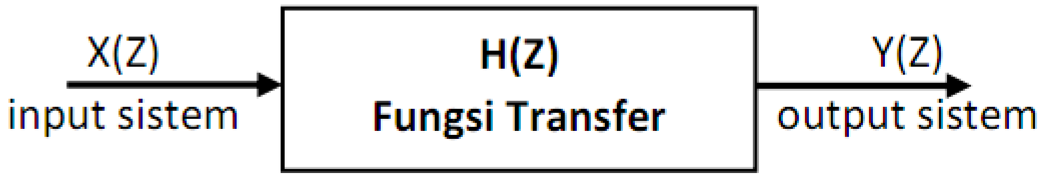
- Fungsi Sistem (Fungsi Transfer) = H(Z) = $\frac{Y(Z)}{X(Z)}$
- Respon impuls = h(n) -> output sistem ketika diberi masukan berupa impuls
- Persamaan selisih (diferensial) = y(n)
- Struktur = diagram blok = berupa gambar rangkaian sistem



Maka, $y(n) = x(n)*h(n)$

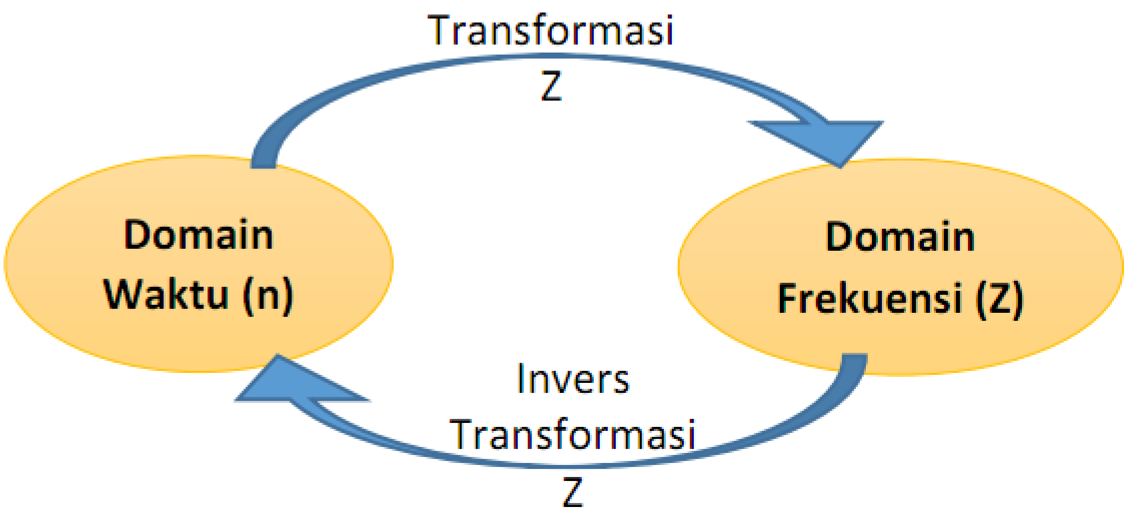
Proses konvolusi

Jika menggunakan Transformasi Z maka akan menjadi:



Maka, $Y(Z) = X(Z).H(Z)$

Proses perkalian sederhana



Fungsi Sistem dari Sistem LTI

$$y(n) = h(n) * x(n) \;\rightarrow\; Y(z) = H(z)X(z) \;\rightarrow\; H(z) = \frac{Y(z)}{X(z)}$$

Respon impuls $h(n) \rightarrow H(z)$ **Fungsi sistem**

Persamaan beda dari sistem LTI :

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

$$Y(z) = -\sum_{k=1}^{N} a_k Y(z) z^{-k} + \sum_{k=0}^{M} b_k X(z) z^{-k}$$

$$Y(z) = -\sum_{k=1}^{N} a_k Y(z) z^{-k} + \sum_{k=0}^{M} b_k X(z) z^{-k}$$

$$Y(z)[1+\sum_{k=1}^{N} a_k z^{-k}] = X(z)\sum_{k=0}^{M} b_k z^{-k}$$

$$\frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}} = H(z)$$

Fungsi sistem rasional

$$\frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} = H(z)$$

Pole-Zero system

Hal khusus I : $a_k = 0, \quad 1 \leq k \leq N$

$$H(z) = \sum_{k=0}^{M} b_k z^{-k} = \frac{1}{z^M} \sum_{k=0}^{M} b_k z^{M-k}$$

All-zero system

Hal khusus II : $b_k = 0, \quad 1 \leq k \leq M$

$$H(z) = \frac{b_o}{1 + \sum_{k=1}^{N} a_k z^{-k}} = \frac{b_o}{\sum_{k=0}^{N} a_k z^{-k}} \qquad a_o = 1$$

All-pole system

- **Contoh 1:**

Tentukan fungsi sistem dan respon impuls sistem LTI :

$$y(n) = \frac{1}{2}y(n-1) + 2x(n)$$

Jawab:

$$Y(z) = \frac{1}{2}z^{-1}Y(z) + 2X(z)$$

$$Y(z)(1 - \frac{1}{2}z^{-1}) = 2X(z)$$

$$H(z) = \frac{2}{1 - \frac{1}{2}z^{-1}} \longrightarrow \boxed{h(n) = 2\left(\frac{1}{2}\right)^n u(n)}$$

- **Contoh 2:**

Tentukan respon impuls dari suatu sistem LTI (Linear Time Invariant) yang dinyatakan oleh persamaan beda/selisih :

$$y(n) + 3y(n-1) + y(n-2) = 4{,}5x(n) + 9{,}5x(n-1)$$

Jawab:

$$Y(z) + 3z^{-1}Y(z) + 2z^{-2}Y(z) = 4{,}5X(z) + 9{,}5z^{-1}X(z)$$

$$Y(z)(1 + 3z^{-1} + 2z^{-2}) = X(z)(4{,}5 + 9{,}5z^{-1})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{4{,}5 + 9{,}5z^{-1}}{1 + 3z^{-1} + 2z^{-2}}$$

$$H(z) = \frac{4{,}5 + 9{,}5z^{-1}}{1 + 3z^{-1} + 2z^{-2}} \Longrightarrow \frac{H(z)}{z} = \frac{4{,}5z + 9{,}5}{z^2 + 3z + 2}$$

$$\frac{H(z)}{z} = \frac{A_1}{z+1} + \frac{A_2}{z+2} = \frac{5}{z+1} - \frac{0{,}5}{z+2}$$

$$H(z) = \frac{5}{1-(-1)z^{-1}} - \frac{0{,}5}{1-(-2)z^{-1}}$$

$$h(n) = [5(-1)^n - 0{,}5(-2)^n]u(n)$$

- **Contoh 3:**

Tentukan output dari suatu sistem LTI (Linear Time Invariant) yang dinyatakan oleh persamaan beda :

$$y(n)+3y(n-1)+y(n-2)=4{,}5x(n)+9{,}5x(n-1)$$

$$y(-1)=0 \qquad y(-2)=0$$

dan mendapat input x(n) = $(-3)^n u(n)$ ⟹ $y(n)=y_{zs}(n)$

Jawab:

$$\mathrm{Y(z)} + 3\mathrm{z}^{-1}\mathrm{Y(z)} + 2\mathrm{z}^{-2}\mathrm{Y(z)} = 4{,}5\mathrm{X(z)} + 9{,}5\mathrm{z}^{-1}\mathrm{X(z)}$$

$$\mathrm{Y(z)}(1 + 3\mathrm{z}^{-1} + 2\mathrm{z}^{-2}) = \mathrm{X(z)}(4{,}5 + 9{,}5\mathrm{z}^{-1})$$

$$Y(z)(1+3z^{-1}+2z^{-2})=(4{,}5+9{,}5z^{-1})X(z)$$

$$x(n)=(-3)^n u(n) \quad \rightarrow \quad X(z)=\frac{1}{1-(-3)z^{-1}}=\frac{1}{1+3z^{-1}}$$

$$Y(z)(1+3z^{-1}+2z^{-2})=(4{,}5+9{,}5z^{-1})\frac{1}{1+3z^{-1}}$$

$$Y(z)=\frac{(4{,}5+9{,}5z^{-1})}{(1+3z^{-1}+2z^{-2})(1+3z^{-1})}$$

$$\frac{Y(z)}{z}=\frac{z^2(4{,}5+9{,}5z^{-1})}{z^3(1+3z^{-1}+2z^{-2})(1+3z^{-1})}$$

$$\frac{Y(z)}{z}=\frac{(4{,}5z^2+9{,}5z)}{(z^2+3z+2)(z+3)}=\frac{(4{,}5z^2+9{,}5z)}{(z+1)(z+2)(z+3)}$$

$$\frac{Y(z)}{z} = \frac{(4{,}5z^2 + 9{,}5z)}{(z^2 + 3z + 2)(z+3)} = \frac{(4{,}5z^2 + 9{,}5z)}{(z+1)(z+2)(z+3)}$$

$$\frac{(4{,}5z^2 + 9{,}5z)}{(z+1)(z+2)(z+3)} = \frac{A_1}{(z+1)} + \frac{A_2}{(z+2)} + \frac{A_3}{(z+3)}$$

$$\frac{Y(z)}{z} = \frac{A_1(z^2 + 5z + 6) + A_2(z^2 + 4z + 3) + A_3(z^2 + 3z + 2)}{(z+1)(z+2)(z+3)}$$

$$\begin{aligned} A_1 + A_2 + A_3 &= 4{,}5 \\ 5A_1 + 4A_2 + 3A_3 &= 9{,}5 \\ 6A_1 + 3A_2 + 2A_3 &= 0 \end{aligned}$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 5 & 4 & 3 \\ 6 & 3 & 2 \end{vmatrix} = 2$$

$$A_1 = \frac{\begin{vmatrix} 4,5 & 1 & 1 \\ 9,5 & 4 & 3 \\ 0 & 3 & 2 \end{vmatrix}}{D} = \frac{-5}{2} = -2,5$$

$$A_2 = \frac{\begin{vmatrix} 1 & 4,5 & 1 \\ 5 & 9,5 & 3 \\ 6 & 0 & 2 \end{vmatrix}}{D} = \frac{2}{2} = 1$$

$$-2,5 + 1 + A_3 = 4,5 \quad \rightarrow \quad A_3 = 6$$

$$\frac{Y(z)}{z} = \frac{-2,5}{(z+1)} + \frac{1}{(z+2)} + \frac{6}{(z+3)}$$

$$Y(z) = \frac{-2,5}{(1+z^{-1})} + \frac{1}{(1+2z^{-1})} + \frac{6}{(1+3z^{-1})}$$

$$y_{zs}(n) = [-2,5(-1)^n + (-2)^2 + 6(-3)^n]u(n)$$

Contoh 4:

Tentukan zero-state response dari suatu sistem LTI yang mendapat input x(n) = u(n) dan dinyatakan oleh persamaan beda :

$$y(n)+6y(n-1)+8y(n-2)=5x(n)-28x(n-1)+8x(n-2)$$

Jawab:

$$Y(z)+6z^{-1}Y(z)+8z^{-2}Y(z)=5X(z)-28z^{-1}X(z)+8z^{-2}X(z)$$

$$X(z)=\frac{1}{1-z^{-1}} \Longrightarrow Y(z)=\frac{(5-28z^{-1}+8z^{-2})}{1+6z^{-1}+8z^{-2}}\frac{1}{1-z^{-1}}$$

$$\frac{Y(z)}{z}=\frac{(5z^2-28z+8)}{(z^2+6z+8)(z-1)}=\frac{A_1}{z+2}+\frac{A_2}{z+4}+\frac{A_3}{z-1}$$

$$\frac{Y(z)}{z}=\frac{(5z^2-28z+8)}{(z+2)(z+4)(z-1)}=\frac{A_1}{z+2}+\frac{A_2}{z+4}+\frac{A_3}{z-1}$$

$$A_1=\frac{(z+2)Y(z)}{z}=\left.\frac{5z^2-28z+8}{(z+4)(z-1)}\right|_{z=-2}=\frac{20+56+8}{(2)(-3)}=\frac{84}{-6}=-14$$

$$A_2=\frac{(z+4)Y(z)}{z}=\left.\frac{5z^2-28z+8}{(z+2)(z-1)}\right|_{z=-4}=\frac{80+112+8}{(-2)(-5)}=\frac{200}{10}=20$$

$$A_3=\frac{(z-1)Y(z)}{z}=\left.\frac{5z^2-28z+8}{(z+2)(z+4)}\right|_{z=1}=\frac{5-28+8}{(3)(5)}=\frac{-15}{15}=-1$$

$$\frac{Y(z)}{z}=\frac{-14}{z+2}+\frac{20}{z+4}+\frac{-1}{z-1}\ \Longrightarrow\ Y(z)=\frac{-14}{1+2z^{-1}}+\frac{20}{1+4z^{-1}}+\frac{-1}{1-z^{-1}}$$

$$y_{zs}(n)=[-14(-2)^n+20(-4)^n-1]\,u(n)$$

- **Contoh 5:**

Tentukan output dari suatu sistem LTI (Linear Time Invariant) yang dinyatakan oleh persamaan beda :

$$y(n)+3y(n-1)+y(n-2)=4{,}5x(n)+9{,}5x(n-1)$$

$$y(-1)=-8{,}5 \qquad y(-2)=7{,}5$$

dengan input x(n) = 0 ⟹ $y(n)=y_{zi}(n)$

Jawab:

$$Y^+(z)+3z^{-1}[Y^+(z)+y(-1)z]$$

$$+2z^{-2}[Y^+(z)+y(-1)z+y(-2)z^2]=0$$

$$\frac{Y^+(z)}{z} = \frac{10{,}5z+17}{z^2+3z+2} = \frac{10{,}5z+17}{(z+1)(z+2)} = \frac{A_1}{z+1} + \frac{A_2}{z+2}$$

$$A_1 = \frac{(z+1)Y^+(z)}{z} = \left.\frac{10{,}5z+17}{z+2}\right|_{z=-1} = \frac{6{,}5}{1} = 6{,}5$$

$$A_2 = \frac{(z+2)Y^+(z)}{z} = \left.\frac{10{,}5z+17}{z+1}\right|_{z=-2} = \frac{-4}{-1} = 4$$

$$Y^+(z) = \frac{6{,}5z}{z+1} + \frac{4z}{z+2} = \frac{6{,}5}{1+z^{-1}} + \frac{4}{1+2z^{-1}}$$

$$y_{zi}(n) = 6{,}5(-1)^n + 4(-2)^n$$

- **Contoh 6:**

Tentukan output dari suatu sistem LTI yang mendapat input x(n) = u(n) dan dinyatakan oleh persamaan beda :

$$y(n)+6y(n-1)+8y(n-2)=5x(n)-28x(n-1)+8x(n-2)$$

$$y(-1)=-4 \quad y(-2)=3$$

Jawab:

$$Y^{+}(z)+6z^{-1}[Y^{+}(z)+y(-1)z]$$

$$+8z^{-2}[Y^{+}(z)+y(-1)z+y(-2)z^{2}]=5X^{+}(z)$$

$$-28z^{-1}[X^{+}(z)+x(-1)z]+8z^{-2}[X^{+}(z)+x(-1)z+x(-2)z^{2}]$$

$$Y^{+}(z)[1+6z^{-1}+8z^{-2}]-24-32z^{-1}+24$$

$$=X^{+}(z)[5-28z^{-1}+8z^{-2}]$$

$$Y^{+}(z)[1+6z^{-1}+8z^{-2}]-24-32z^{-1}+24$$

$$= X^{+}(z)[5-28z^{-1}+8z^{-2}]$$

$$Y^{+}(z)[1+6z^{-1}+8z^{-2}] = 32z^{-1}+\frac{5-28z^{-1}+8z^{-2}}{1-z^{-1}}$$

$$Y^{+}(z) = \frac{32z^{-1}-32z^{-2}+5-28z^{-1}+8z^{-2}}{(1+6z^{-1}+8z^{-2})(1-z^{-1})}$$

$$Y^{+}(z) = \frac{5+4z^{-1}-24z^{-2}}{(1+6z^{-1}+8z^{-2})(1-z^{-1})}$$

$$\frac{Y^{+}(z)}{z} = \frac{5z^{2}+4z-24}{(z^{2}+6z+8)(z-1)}$$

$$\frac{Y^{+}(z)}{z} = \frac{5z^2 + 4z - 24}{(z^2 + 6z + 8)(z-1)} = \frac{5z^2 + 4z - 24}{(z+2)(z+4)(z-1)}$$

$$\frac{5z^2 - 4z - 24}{(z+2)(z+4)(z-1)} = \frac{A_1}{z-1} + \frac{A_2}{z+2} + \frac{A_3}{z+4}$$

$$A_1 = \left.\frac{5z^2 + 4z - 24}{(z+2)(z+4)}\right|_{z=1} = \frac{5 + 4 - 24}{(3)(5)} = \frac{-15}{15} = -1$$

$$A_2 = \left.\frac{5z^2 + 4z - 24}{(z+4)(z-1)}\right|_{z=-2} = \frac{20 - 8 - 24}{(2)(-3)} = \frac{-12}{-6} = 2$$

$$A_3 = \left.\frac{5z^2 + 4z - 24}{(z+2)(z-1)}\right|_{z=-4} = \frac{80 - 16 - 24}{(-2)(-5)} = \frac{40}{10} = 4$$

$$\frac{Y^+}{z} = \frac{-1}{z-1} + \frac{2}{z+2} + \frac{4}{z+4}$$

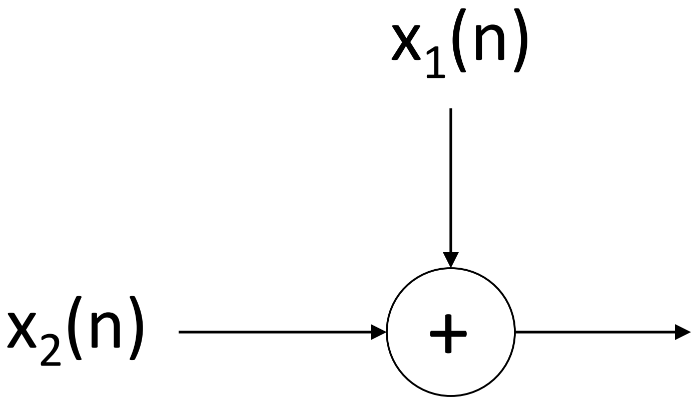
$$Y^+ = \frac{-1}{1-z^{-1}} + \frac{2}{1+2z^{-1}} + \frac{4}{1+4z^{-1}}$$

$$y(n) = [-1 + 2(-2)^n + 4(-4)^2]u(n)$$

Representasi Diagram Blok

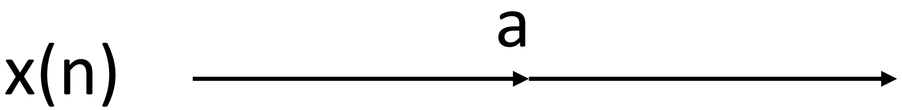
- Penjumlah (adder)
- Pengali dengan konstanta (constant muliplier)
- Pengali sinyal (signal multiplier)
- Elemen tunda (unit delay element)

Adder :



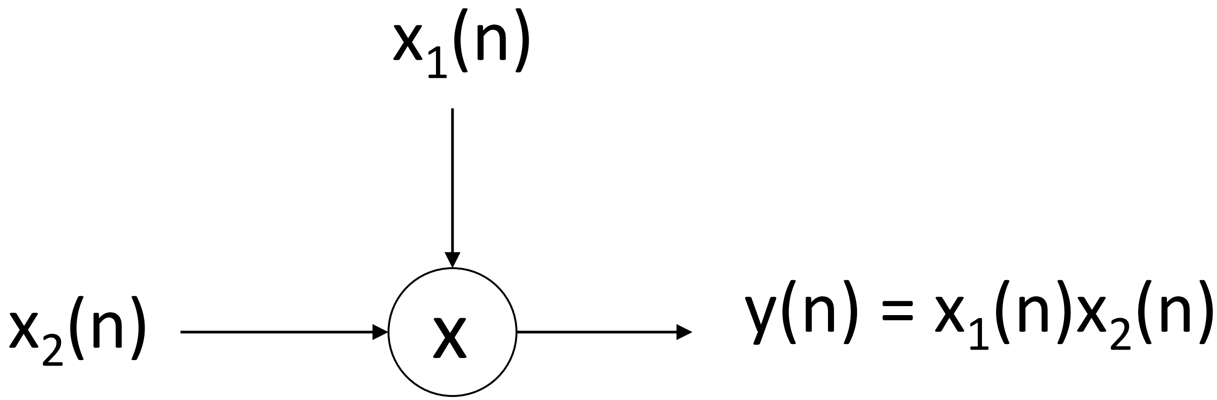
$$y(n) = x_1(n) + x_2(n)$$

Constant multiplier :

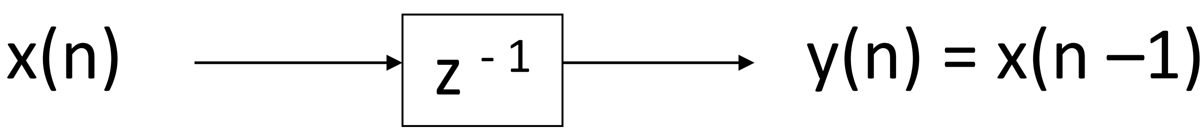


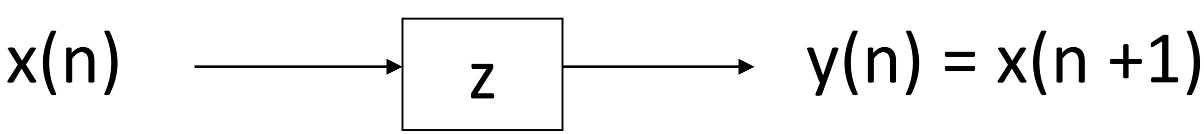
$$y(n) = a\,x(n)$$

Signal multiplier :



Unit delay element :



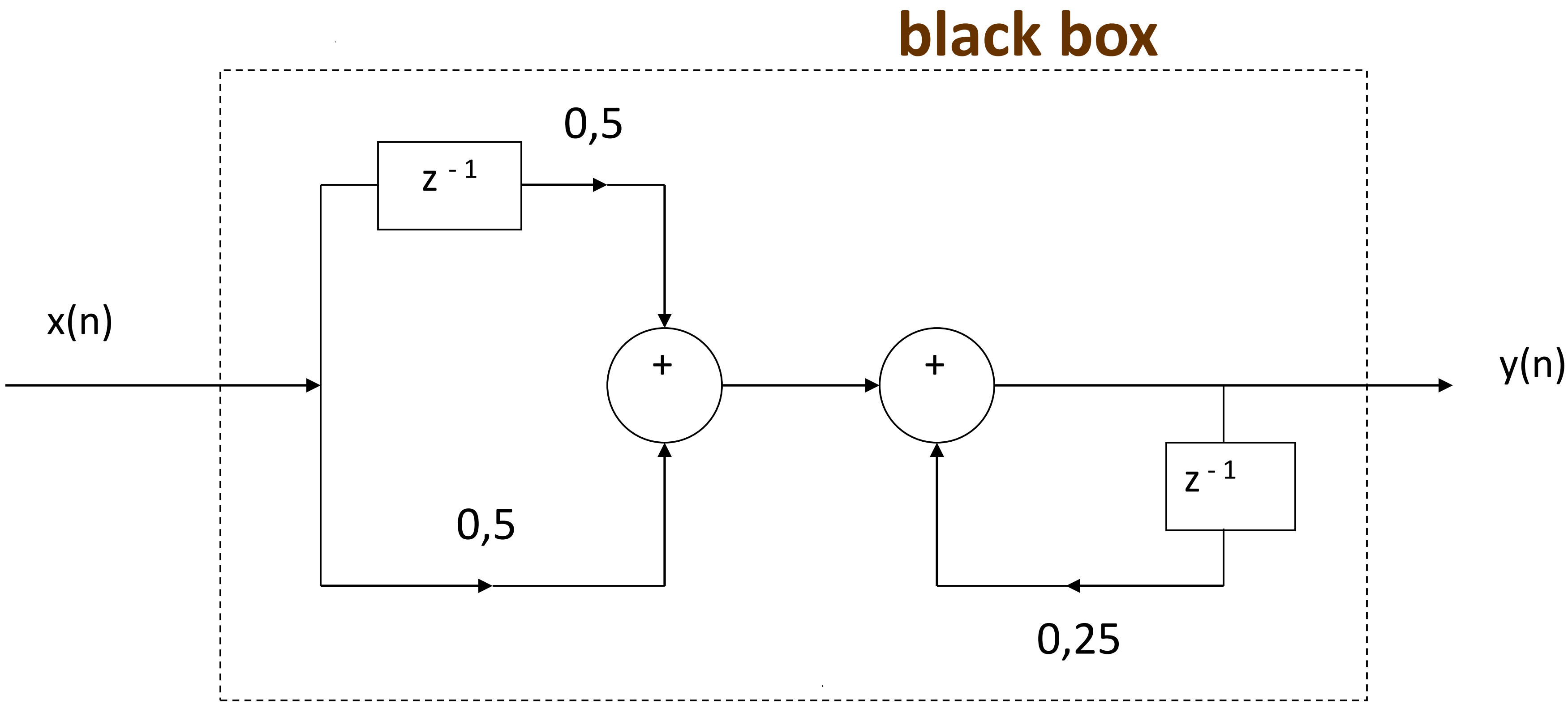


- ## Contoh 9:

Buat diagram blok dari sistem waktu diskrit dimana :

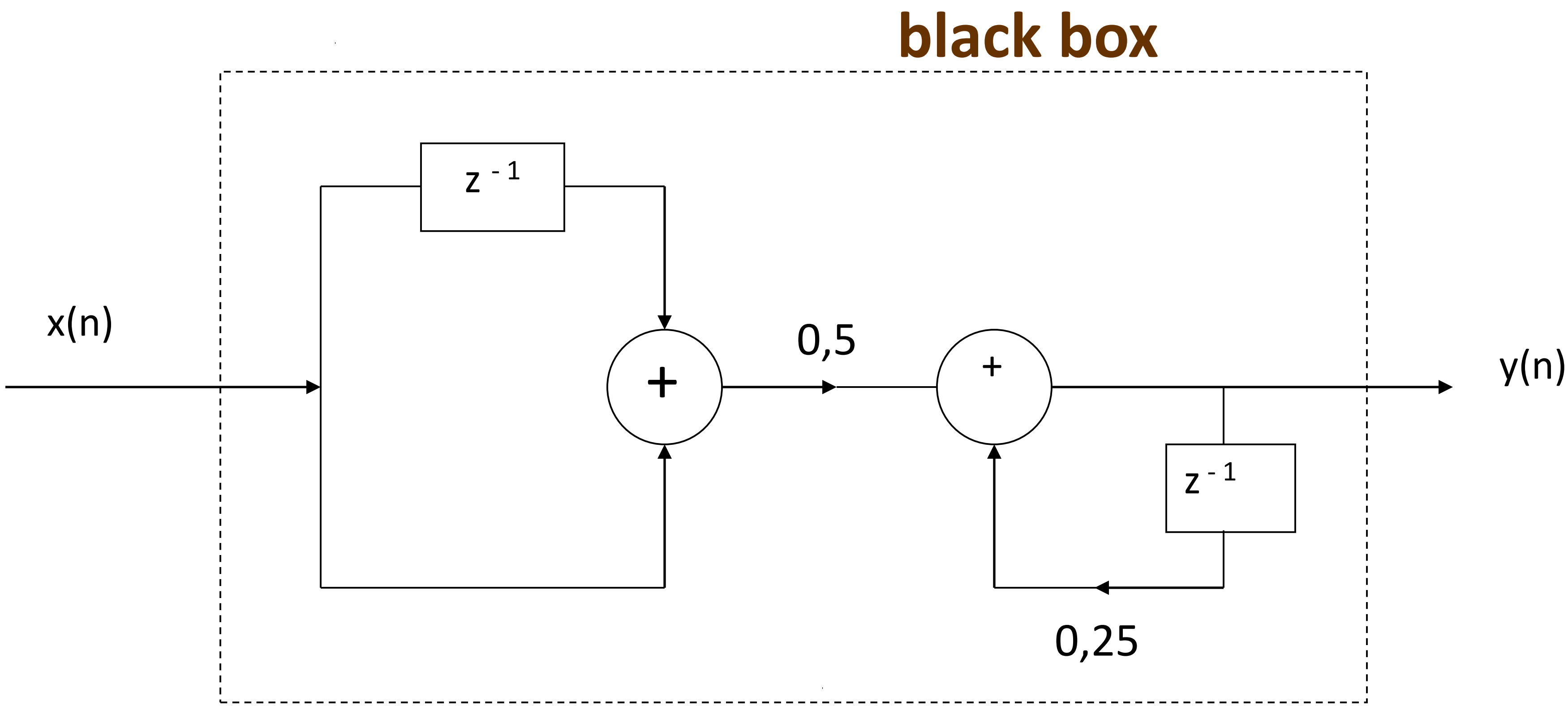
$$y(n) = \frac{1}{4}y(n-1) + \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$$

<u>Jawab :</u>

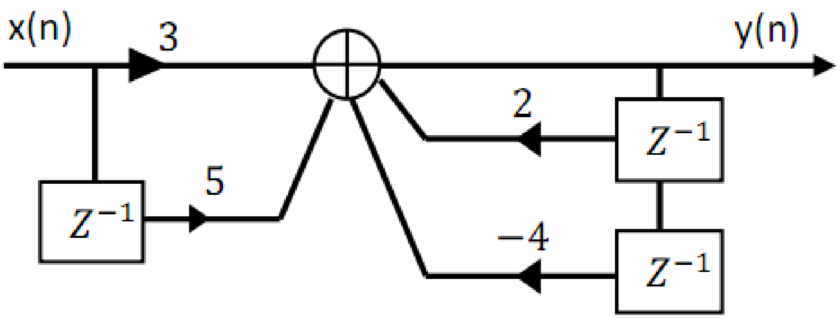


$$y(n) = \frac{1}{4}y(n-1) + \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$$

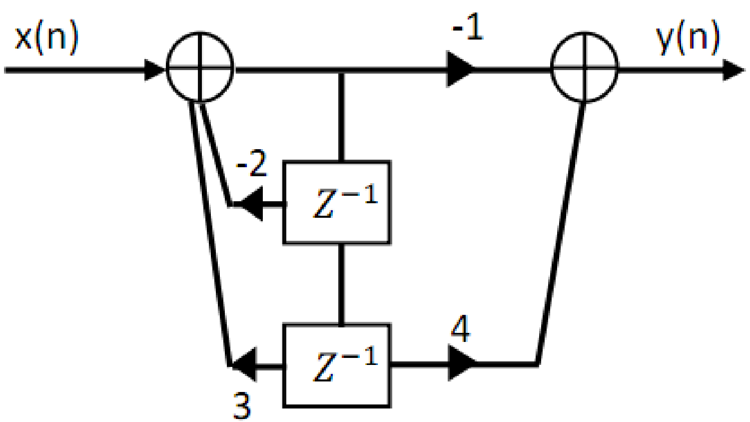
$$y(n) = \frac{1}{4}y(n-1) + \frac{1}{2}[x(n) + x(n-1)]$$



$$Fungsi\ Transfer\ [H(Z)] = \frac{\sum forward}{1 - \sum loop}$$



Maka, $H(Z) = \dfrac{3+5Z^{-1}}{1-(2Z^{-1}-4Z^{-2})}$



Maka, $H(Z) = \dfrac{-1+4Z^{-2}}{1-(-2Z^{-1}+3Z^{-2})}$

Klasifikasi Sistem

- Sistem statik dan dinamik
- Time-invariant & time-variant system
- Sistem linier dan sistem nonlinier
- Sistem kausal dan sistem nonkausal
- Sistem stabil dan sistim tak stabil

Sistem Statik (memoryless) :

- **Output pada setiap saat hanya tergantung input pada saat yang sama**
- **Tidak tergantung input pada saat yang lalu atau saat yang akan datang**

$$y(n) = a\,x(n)$$

$$y(n) = n\,x(n) + b\,x^3(n)$$

$$y(n) = T[x(n), n]$$

Sistem Dinamik :

- **Outputnya selain tergantung pada input saat yang sama juga tergantung input pada saat yang lalu atau saat yang akan datang**

$$y(n) = x(n) + 3x(n-1) \longrightarrow$$ Memori terbatas

$$y(n) = \sum_{k=0}^{n} x(n-k) \longrightarrow$$ Memori terbatas

$$y(n) = \sum_{k=0}^{\propto} x(n-k) \longrightarrow$$ Memori tak terbatas

Sistem Time-Invariant (shift-invariant) :

- **Hubungan antara input dan output tidak tergantung pada waktu**

$$y(n) = T[x(n)] \longrightarrow y(n-k) = T[x(n-k)]$$

Umumnya : $y(n,k) = T[x(n-k)]$

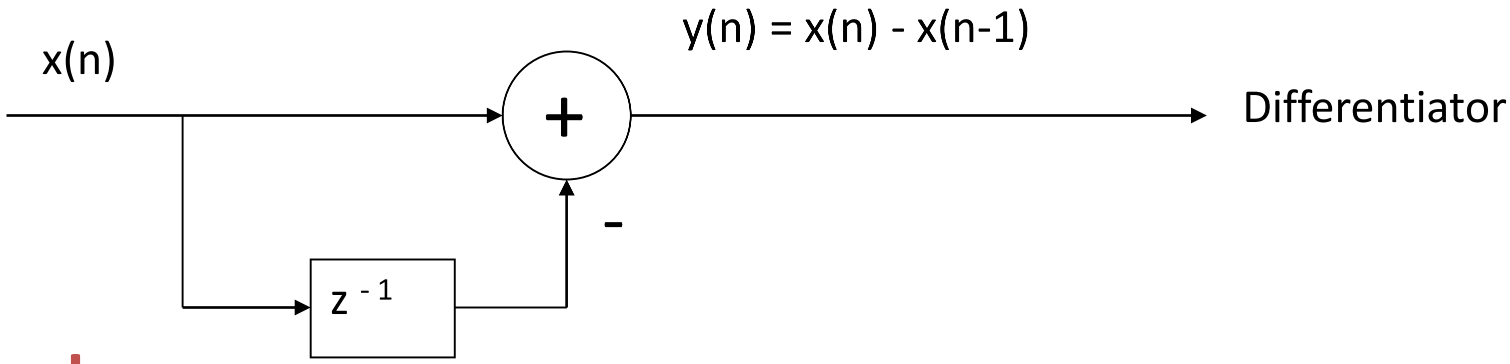
$$y(n,k) = y(n-k) \longrightarrow \text{Time-invariant}$$

$$y(n,k) \neq y(n-k) \longrightarrow \text{Time-variant}$$

- **Contoh 10:**

Tentukan apakah sistem-sistem di bawah ini time-invariant atau time-variant

a)



Jawab :

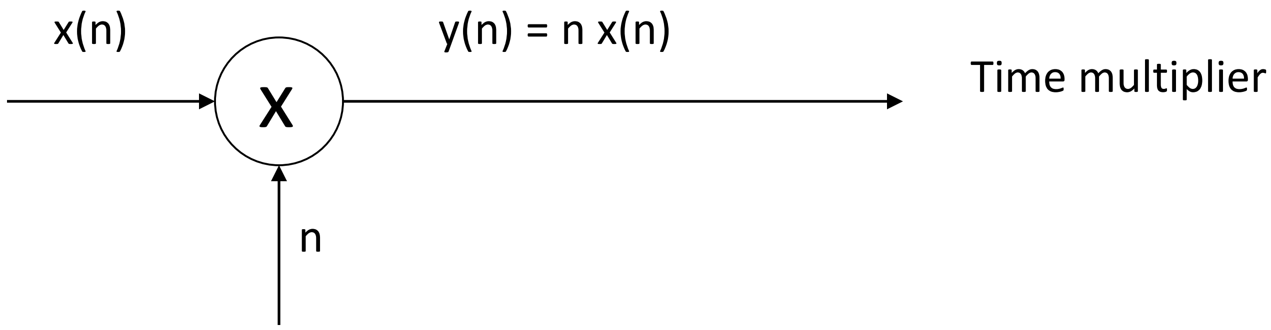
$$y(n) = T[x(n)] = x(n) - x(n-1)$$

$$y(n,k) = T[x(n-k)] = x(n-k) - x(n-k-1)$$

$$y(n-k) = x(n-k) - x(n-k-1)$$

$$y(n,k) = y(n-k)$$ → **Time-invariant**

b)



Jawab :

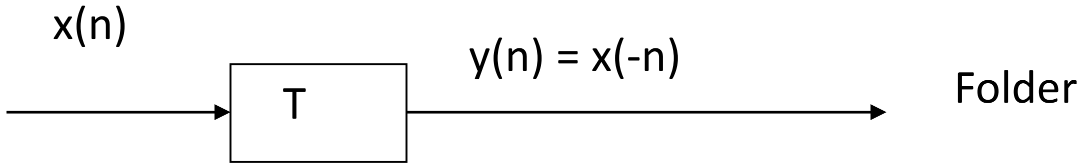
$$y(n) = T[x(n)] = nx(n)$$

$$y(n,k) = T[x(n-k)] = nx(n-k)$$

$$y(n-k) = (n-k)x(n-k) = nx(n-k) - kx(n-k)$$

$$y(n,k) \neq y(n-k)$$ ⟹ **Time-variant**

c)



Jawab :

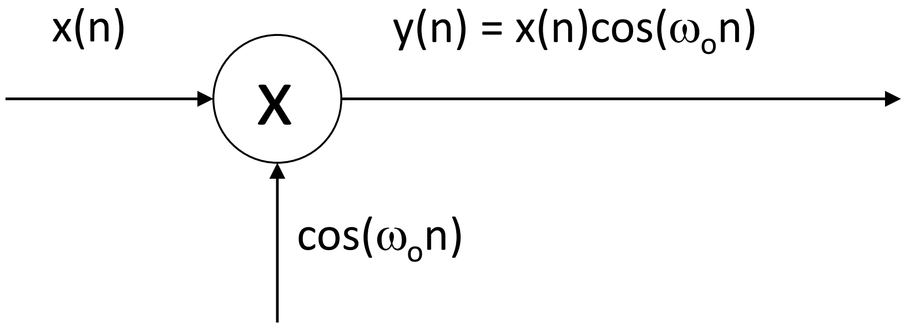
$$y(n) = T[x(n)] = x(-n)$$

$$y(n,k) = T[x(n-k)] = x(-n-k)$$

$$y(n-k) = x[-(n-k)] = x(-n+k)$$

$y(n,k) \neq y(n-k)$ ⟹ **Time-variant**

d)



Modulator

Jawab :

$$y(n) = T[x(n)] = x(n)\cos(\omega_o n)$$

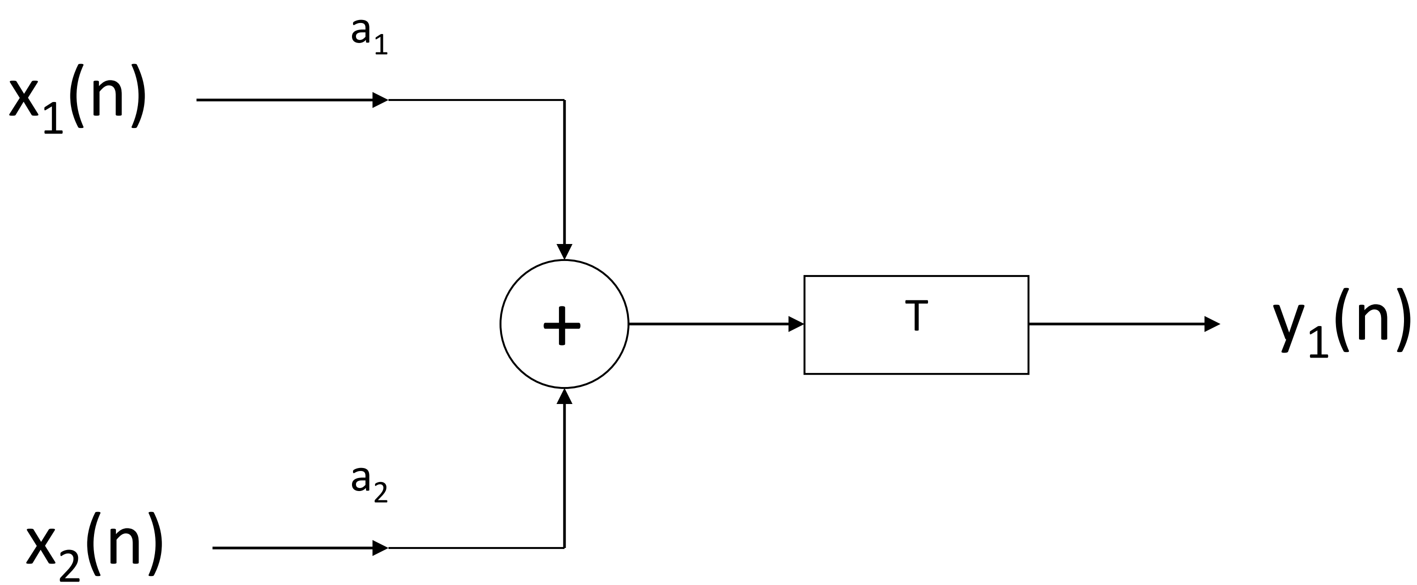
$$y(n,k) = T[x(n-k)] = x(n-k)\cos(\omega_o n)$$

$$y(n-k) = x(n-k)\cos[\omega_o (n-k)]$$

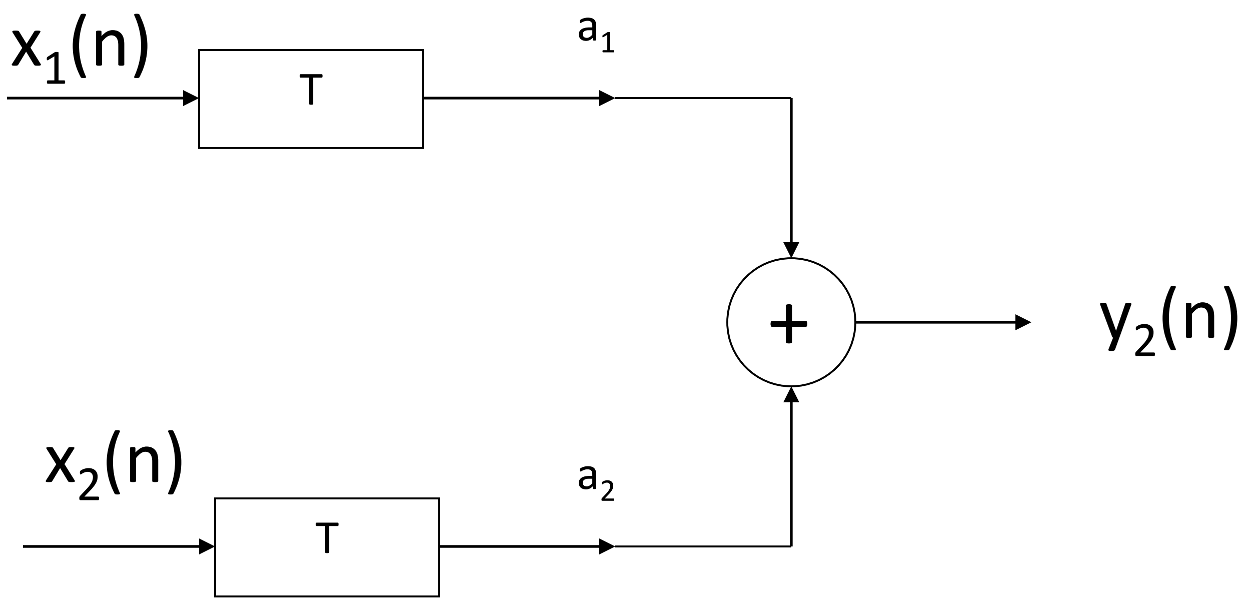
$$y(n,k) \neq y(n-k)$$ ⟶ **Time-variant**

Sistem Linier :

- **Prinsip superposisi berlaku**



$$y_1(n) = T[a_1 x_1(n) + a_2 x_2(n)]$$



$$y_2(n) = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

$$y_1(n) = y_2(n)$$ ⟹ **Linier**

- **Contoh 11:**

Tentukan apakah sistem-sistem di bawah ini linier atau nonlinier

$$a) \quad y(n) = nx(n)$$

$$b) \quad y(n) = x(n^2)$$

$$c) \quad y(n) = x^2(n)$$

$$d) \quad y(n) = Ax(n) + B$$

Sistem Kausal :

- **Outputnya hanya tergantung pada input sekarang dan input yang lalu**
  - **x(n), x(n-1), x(n-2), …..**
- **Outputnya tidak tergantung pada input yang lalu**
  - **x(n+1), x(n+2), …..**

$$y(n) = F[x(n), x(n-1), x(n-2), \cdots]$$

- **Contoh 12:**

Tentukan kausalitas dari sistem-sistem di bawah ini :

$$a)\quad y(n) = x(n) - x(n-1)$$

$$b)\quad y(n) = \sum_{k=-\infty}^{n} x(k)$$

a, b dan c kausal

$$c)\quad y(n) = a\,x(n-k)$$

$$d)\quad y(n) = x(n) + 3\,x(n+4)$$

$$e)\quad y(n) = x(n^2)$$

d, e dan f nonkausal

$$f)\quad y(n) = x(2n)$$

$$g)\quad y(n) = x(-n)$$

g kausal

Sistem Stabil :

- **Setiap input yang terbatas (bounded input) akan menghasilkan output yang terbatas (bounded output) → BIBO**

$$|x(n)| \le M_x \le \propto \quad \longrightarrow \quad |y(n)| \le M_y \le \propto$$

Contoh 13:

Tentukan kestabilan dari sistem di bawah ini

$$y(n) = y^2(n-1) + x(n) \qquad y(-1) = 0$$

bila mendapat input x(n) = C δ(n), 1 < C < ∝

Jawab :

$$y(0) = C$$

$$y(1) = C^2$$

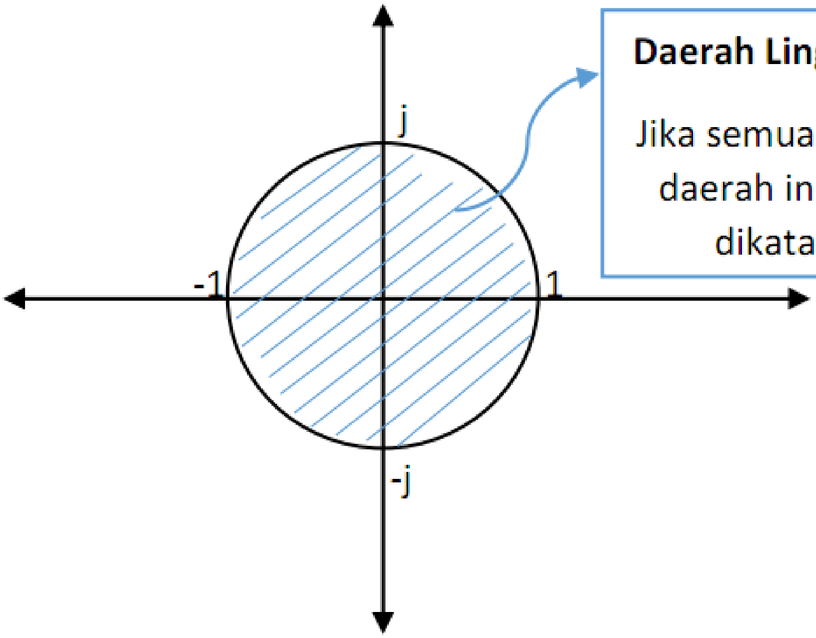
$$y(2) = C^4$$

$$y(n) = C^{2n} \longrightarrow$$ **Tidak stabil**

Kestabilan sistem dapat diketahui melalui pole dan zero dari Fungsi Transfer

Pole : harga-harga $z = p_i$ yang menyebabkan $H(z) = \infty$

Zero : harga-harga $z = z_i$ yang menyebabkan $H(z) = 0$



Daerah Lingkaran Satuan.

Jika semua pole berada di daerah ini maka sistem dikatakan stabil.

Jika pole berada tepat di garis lingkaran satuan maka disebut **stabil marginal**

- **Contoh 9:**

Tentukan pole dan zero dari $$H(z)=\frac{2-1{,}5z^{-1}}{1-1{,}5z^{-1}+0{,}5z^{-2}}$$

Jawab:

$$H(z)=2\,\frac{z^{-1}}{z^{-2}}\,\frac{z-0{,}75}{z^2-1{,}5z+0{,}5}$$

$$=2\,z\frac{z-0{,}75}{(z-1)(z-0{,}5)}=\frac{2z(z-0{,}75)}{(z-1)(z-0{,}5)}$$

$$\therefore\quad Zero:\; z_1=0 \quad z_2=0{,}75$$
$$Pole:\; p_1=1 \quad p_2=0{,}5$$

→ **STABIL**

Contoh 10:

Tentukan pole dan zero dari $H(z) = \dfrac{1+z^{-1}}{1-z^{-1}+0{,}5z^{-2}}$

Jawab:

$$H(z) = \frac{z(z+1)}{z^2 - z + 0{,}5}$$

$$= \frac{z(z+1)}{[z-(0{,}5+j0{,}5)][z-(0{,}5-j0{,}5)]}$$

STABIL ???

$$\therefore \; Zero: \; z_1 = 0 \quad z_2 = 1$$

$$Pole: \; p_1 = 0{,}5 + j0{,}5 \qquad p_2 = 0{,}5 - j0{,}5 \rightarrow p_1 = p_2^{*}$$

Contoh 11:

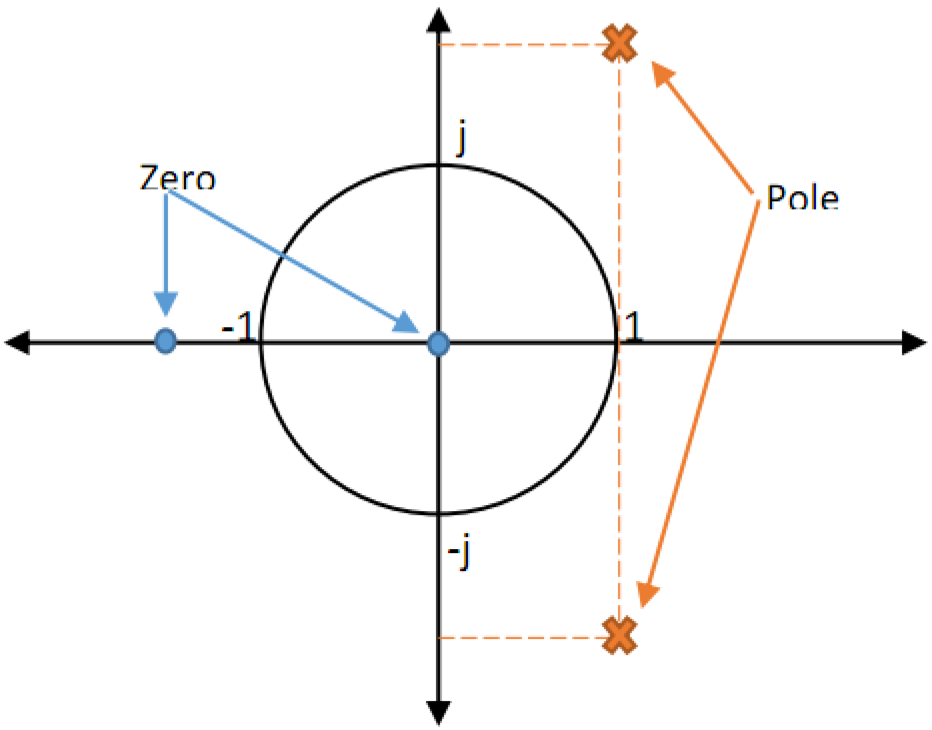
$$H(Z) = \frac{3 + 5Z^{-1}}{1 - (2Z^{-1} - 4Z^{-2})} = \frac{3Z^2 + 5Z}{Z^2 - 2Z + 4}$$

$3Z^2 + 5Z$ → Pembilang, Menentukan nilai zero

$Z^2 - 2Z + 4$ → Penyebut, Menentukan nilai pole

Zero (ditandai dengan o)	Pole (ditandai dengan x)
$3Z^2 + 5Z = 0$ $Z\,(3Z + 5) = 0$ Zero$_1$ = 0 , Zero$_2$ = -5/3	$Z^2 - 2Z + 4 = 0$ Persamaan tersebut tidak bias difaktorkan langsung, maka untuk mencari akar-akarnya menggunakan persamaan: $Z_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}, \quad Z_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$ Dari persamaan tersebut di atas, didapat: Pole$_1$ = 1 + j1,732 , Pole$_2$ = 1 – j1,732 Ingat, bahwa $\sqrt{-1} = j$

Plot Nilai Pole dan Zero tersebut ke dalam diagram lingkaran satuan.



STABIL ???

Mengapa pole harus berada di dalam lingkaran satuan sehingga sistem disebut stabil? Perhatikan persamaan Fungsi Transfer berikut ini:

$$H(Z) = \frac{1}{1 - 2Z^{-1}} = \frac{Z}{Z - 2} \quad \rightarrow \quad h(n) = 2^n . U(n)$$

$$\rightarrow \quad nilai\ menjadi\ tak\ hingga\ maka\ sistem\ menjadi\ tidak\ stabil$$

$$H(Z) = \frac{1}{1 - \frac{1}{2}Z^{-1}} = \frac{Z}{Z - \frac{1}{2}} \quad \rightarrow \quad h(n) = \frac{1}{2}^n . U(n)$$

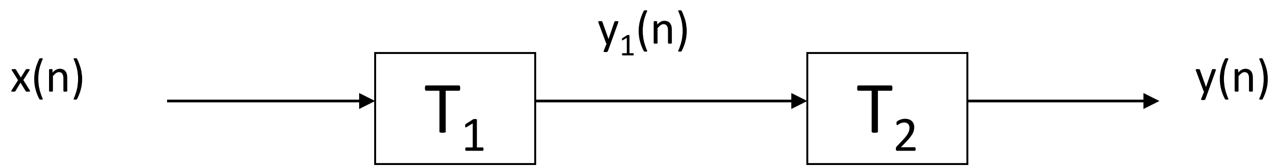
$$\rightarrow \quad nilai\ menjadi\ terhingga, menuju\ nilai\ tertentu, maka\ sistem\ menjadi\ stabil$$

Maka terbukti bahwa jika pole berada di luar lingkaran satuan, sistem menjadi tidak stabil karena keluarannya menjadi tidak terbatas.

Hubungan Antar Sistem

- **Sistem-sistem kecil dapat digabungkan menjadi sistem yang lebih besar**

Hubungan seri



$$y_1(n) = T_1[x(n)]$$

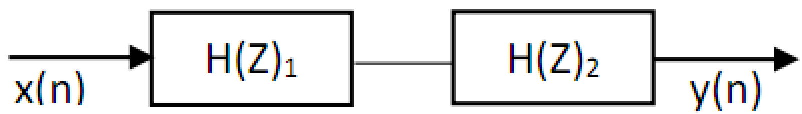
$$y(n) = T_2[y_1(n)] = T_2\{T_1[x(n)]\}$$

$$T_c = T_2 T_1 \quad \rightarrow \quad y(n) = T_c[x(n)]$$

Umumnya : $T_2T_1 \neq T_1T_2$

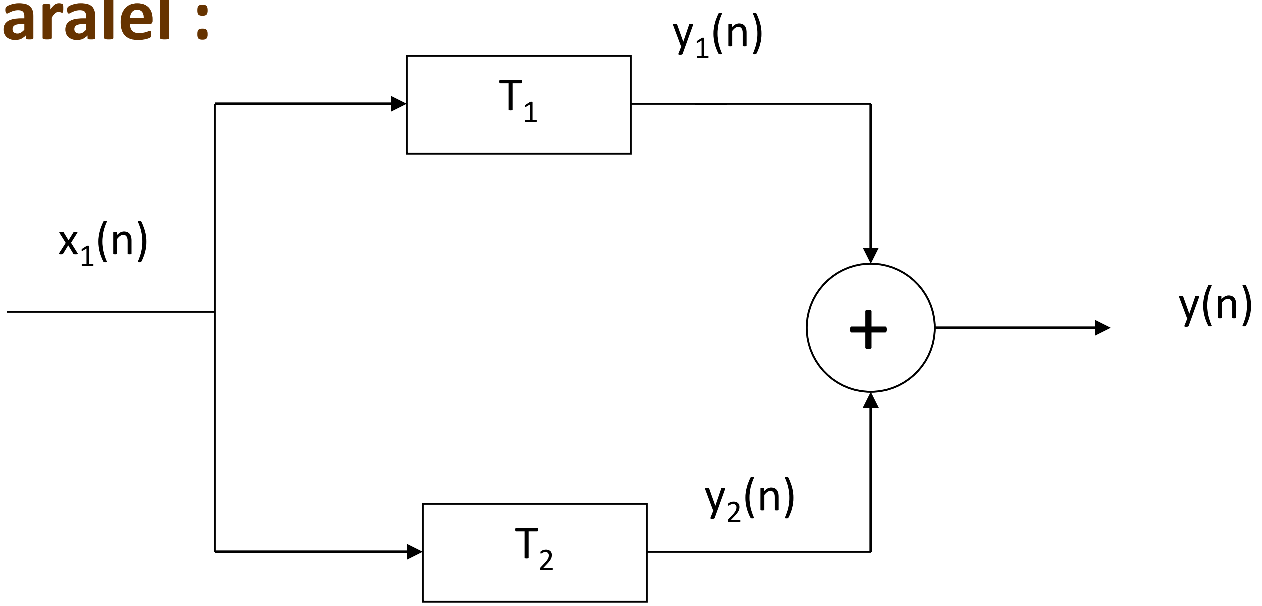
Sistem linier dan time-invariant : $T_2T_1 = T_1T_2$

Pada sistem yang serial (dikalikan), jika terdapat sistem yang tidak stabil namun pole pembuat tidak stabil bisa dieliminasi dengan proses perkalian terhadap sistem yang lainnya maka keseluruhan sistem bisa menjadi stabil.



$H(Z)_1 = \dfrac{z+3}{Z-\frac{1}{2}}$	$H(Z)_2 = \dfrac{z}{Z+3}$	$H(Z)_{Total} = \dfrac{z+3}{Z-\frac{1}{2}} \cdot \dfrac{z}{Z+3} = \dfrac{z}{Z-\frac{1}{2}}$
Stabil	Tidak Stabil	Sistem keseluruhan menjadi stabil

Hubungan paralel :



$$y(n) = y_1(n) + y_2(n) = T_1[x(n)] + T_2[x(n)]$$

$$T_p = (T_1 + T_2)$$

$$y(n) = (T_1 + T_2)[x(n)] = T_p[x(n)]$$

Pada sistem yang parallel (dijumlahkan) maka jika ada salah satu sistem tidak stabil maka keseluruhan sistem menjadi tidak stabil.

Kuis ☺

1. Diketahui input sistem adalah $x(n) = \delta(n) + \delta(n-1)$. Sistem tersebut merupakan sistem LTI yang memiliki keluaran (output) $y(n) = \delta(n) + 2\delta(n-1) + \delta(n-2)$. Tentukan keluaran sistem jika inputnya sebagai berikut:

a. $x(n) = \delta(n) + 2\delta(n-1) + \delta(n-2)$

b. $x(n) = \delta(n-1) - \delta(n-3)$

KUIS ☺

2. Respon impuls suatu sistem adalah $\left(\frac{1}{2}\right)^n . U(n)$. Jika sistem tersebut diberi sinyal masukan $2^n . U(n)$. Maka, tentukanlah persamaan sinyal keluaran dari sistem tersebut !

KUIS ☺

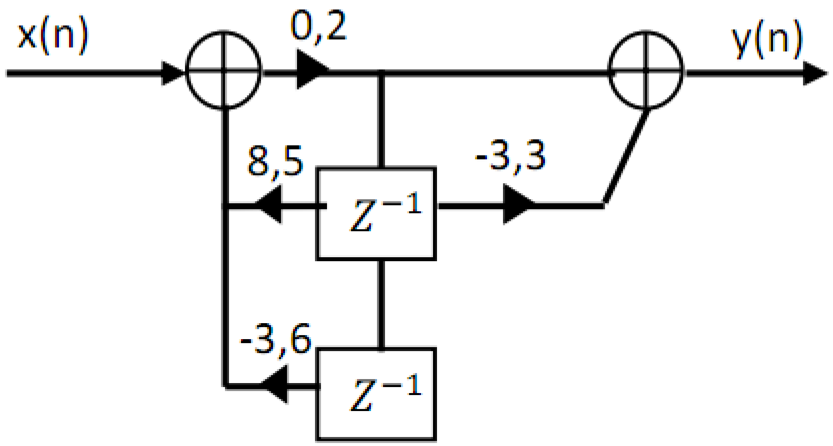
3. Sistem LTI diskrit memiliki fungsi transfer

$$H(Z) = \frac{2 + Z^{-1}}{3 + 5Z^{-1} + 4Z^{-2}}$$

a) Gambarkan struktur realisasinya dg memori besar (3 delay)
b) Gambarkan struktur realisasinya dg memori kecil (2 delay)
c) Tentukan persamaan selisihnya

KUIS ☺

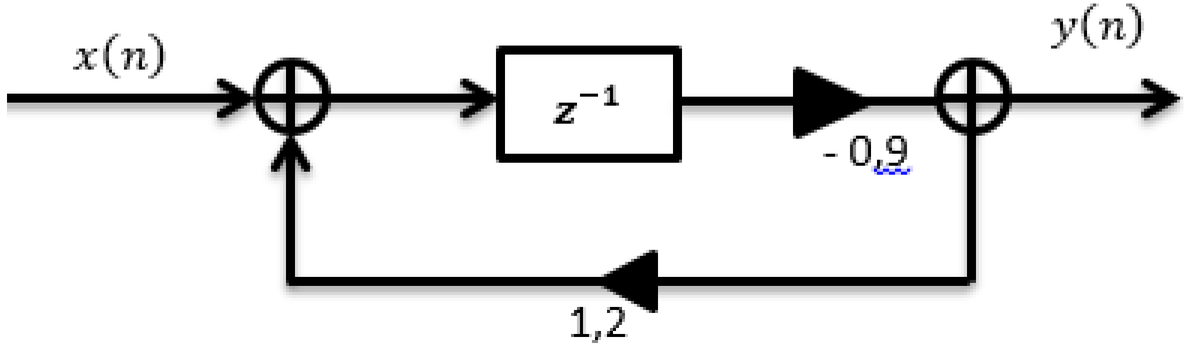
4



a) Tentukan Fungsi Transfer sistem di atas!
b) Tentukan Pole dan Zero dari sistem di atas!
c) Apakah sistem tersebut stabil?

KUIS ☺

5



a) Tentukan Fungsi Transfer sistem di atas!
b) Tentukan Pole dan Zero dari sistem di atas!
c) Apakah sistem tersebut stabil?